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Abstract: This paper deals with the state estimation for a schistosomiasis infection dynamical model described by a continuous non linear system when only the infected human population is measured. The central idea will be studied following two major angles. On the one hand, when all the parameters of the model are supposed to be well known, we will construct a simple observer and a high-gain Luenberger observer based on a canonical controller form and conceived for the nonlinear dynamics where it is implemented.

On the other hand, when the nonlinear uncertain continuous-time system is in a bounded-error context, we will introduce a method for designing a guaranteed interval observer. Numerical simulations are included in order to test the behavior and the performance of the given observers.

Key-words: Nonlinear dynamical systems, Observer, Schistosomiasis.

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Synthèse d'observateur d'un modéle non linéaire de la Bilharziose

Résumé : Un observateur 'grand gain' non-linéaire est mis en uvre pour évaluer l'évolution de dynamique d'une infection de la Bilharziose décrite par un modèle continu non linéaire [1]. On propose un modèle réduit du modèle [1] de la Bilharziose pour construire l'observateur. Des simulations numériques ont été faites pour tester le comportement et la performance de l'observateur proposé.

Mots-clés : Systèmes dynamiques non-linéaires, Observateur, Bilharziose

1 Introduction

Human schistosomiasis is a behavioral and occupational disease associated with poor human hygiene, insanitary animal husbandry and economic activities. Among human parasitic diseases, schistosomiasis ranks second behind malaria second behind malaria as far as the socio-economic and public health importance in tropical and subtropical areas are concerned. Urinary schistosomiasis, caused by the species Schistosoma haematobium, is common in Africa and the Middle East. The main clinical sign of schistosomiasis infection is haematuria itself caused by the depositions of eggs by an adult female's worms through the bladder by urinary intermediary [16]. The most effective form of treatment for infected individuals is the use of the drug praziquantel a drug that kills the worms with high efficiency. Control programs communities often consist on mass chemotherapy possibly supplemented by snail (intermediate host) control. Since school-age children are the heaviest infected group that suffer the most from morbidity and by that are major sources of infection for the community, school targeted chemotherapy can be then an adequate effective approach to control that morbidity. [16], [19].

The spread and persistence of schistosomiasis have made of it one of the most complex host-parasite process to model mathematically because of the different steps of growth of larval assumed by the parasite and the requirement of two host elements (definitive human host and intermediate snail hosts) during their life cycle.

An efficient method to control the schistosmiasis infection that may require relatively little funding is a biological control. Particularly, trematode parasites or competitive snails of the intermediate snail hosts have been proved to be effective in controlling schistosomiasis in the Caribbean area (Pointier and Jourdane, 2000). Actually, Allen and al. proposed a model incorporating this kind of control strategies, and it has been proved that it can be found resistant snail species able to fight against the intermediate host snail species.

Ordinary differential Equations (ODEs) are used to describe population evolution over a continuous time period. Deterministic ODEs are one of the major modeling tools and are used in our case. Symbolically one writes

$$\left\{\begin{array}{lcl} \dot{X}(t) & = & F(X(t)), \\ Y(t) & = & h(X(t)), \end{array}\right. \tag{1}$$

with $X(t) \in \mathbb{R}^n$, $Y(t) \in \mathbb{R}^p$, $p < n$.

If it is possible to have the value of the state at some time t_0 then it is possible to compute $X(t)$ for all $t \geq t_0$ by integrating the differential equation with the initial condition $X(t_0)$. Unfortunately, it is not often possible to measure the whole state at a given time and and by the same way to integrate the differential equation because one does not know the initial condition. One can only have a partial information of the state and this partial information is precisely given by $Y(t)$ the output of the system. Therefore we shall show how to use this partial information $Y(t)$ together with the given model in order to have a reliable estimation of the unmeasurable state variables. A state observer is usually employed, in order to accurately reconstruct the state variables of the dynamical system. In the case of linear systems, the observer design theory developed by Luenberger [13], offers a complete and comprehensive answer to the problem. In the field of nonlinear systems, the nonlinear observer design problem is much more challenging and has received a considerable amount of attention in the literature.

An observer for (1) is a dynamical system

$$\begin{cases} \dot{Z}(t) = \hat{F}(Z(t), Y(t)), \\ \hat{X}(t) = L(Z(t), Y(t)) \end{cases} \tag{2}$$

whose task is state estimation. It is expected to provide a dynamical estimate $\hat{X}(t)$ of the state $X(t)$ of the original system. The output is in general a function of the state variable, that is, $Y(t) = h(X(t))$.
One usually requires at least that $\left|\hat{X}(t) - X(t)\right|$ goes to zero as $t \to \infty$. When the convergence of $\hat{X}(t)$ towards $X(t)$ is exponential, the system (1) is an "exponential observer". More precisely, system (2) is an exponential observer for system (1) if there exists $\lambda > 0$ and $c_0 \geq 0$ such that, for all $t \geq 0$ and for all initial conditions $\left(X(0), \hat{X}(0)\right)$, the corresponding solutions of (1)-(2) satisfy

$$\left|\hat{X}(t) - X(t)\right| \leq e^{-\lambda\, t} \left(\left|\hat{X}(0) - X(0)\right| + c_0\right).$$

The best situation corresponds to the case where $c_0 = 0$. In this situation a good estimate of the real unmeasured state is rapidly obtained. One must notice that we do not need to care about of the initial condition of the observer since the convergence of $\hat{X}(t)$ towards the real state $X(t)$ does not depend on this choice.
We will consider a simple schistosomiasis dynamical model described by:

$$\begin{cases} \dfrac{dX_1}{dt} = -t_{15}\, X_5\, X_1 + r_{12}\, X_2, \\ \dfrac{dX_2}{dt} = t_{15}\, X_5\, X_1 - r_{12}\, X_2, \\ \dfrac{dX_3}{dt} = b_3\, (X_3 + X_4 + X_5) - t_{32}\, X_2\, X_3 - d_3\, X_3 - t_{37}\, X_3\, X_7, \\ \dfrac{dX_4}{dt} = t_{32}\, X_2\, X_3 + t_{37}\, X_3\, X_7 - d_4\, X_4 - r_{54}\, X_4, \\ \dfrac{dX_5}{dt} = r_{54}\, X_4 - d_5\, X_5, \\ \dfrac{dX_6}{dt} = b_6 (X_6 + X_7) - t_{65}\, X_5\, X_6 - d_6\, X_6, \\ \dfrac{dX_7}{dt} = t_{65}\, X_5\, X_6 - d_7 X_7. \end{cases} \tag{3}$$

Where

$X_1(t)$ = the susceptible (uninfected) human population size,
$X_2(t)$ = the infected human population size,
$X_3(t)$ = the susceptible snail host population size,
$X_4(t)$ = the population size of the infected snails which are not yet shedding cercariae,
$X_5(t)$ = the infected and shedding snail population size (shedding population size),
$X_6(t)$ = the susceptible mammal population size,
$X_7(t)$ = the infected mammal population size.

Birth and death rates for the various sub populations are denoted by b_i et d_i, respectively, for $i = 1, 2, ..., 7$. The transmission parameters for the model are:

$$
\begin{array}{lcl}
t_{15} & = & \text{transmission rate from infected snails to uninfected humans,} \\
t_{32} & = & \text{transmission rate from infected humans to uninfected snails,} \\
t_{37} & = & \text{transmission rate from infected mammals to susceptible snail,} \\
t_{65} & = & \text{transmission rate from infected snails to susceptible mammals.}
\end{array}
$$

Also, r_{12} is the rate that infected humans recover and r_{54} denotes the rate that the latent snail population X_4 becomes shedding X_5.
It is assumed for simplicity that $b_3 = d_3 = d_4 = d_5$, and $b_6 = d_6 = d_7$.
We denote the total human population, total snails population and the total mammals population respectively by: $N_H = X_1 + X_2$, $N_S = X_3 + X_4 + X_5$, and $N_M = X_6 + X_7$. All these total populations are constant thanks to the assumptions on b_i and d_i. We introduce the proportions of the snails and mammals subpopulations: $x_i = \dfrac{X_i}{N_S}$ for $i = 3, 4, 5$, and $x_i = \dfrac{X_i}{N_M}$ for $i = 6, 7$.
Using the number of infected humans and the proportions of the other subpopulations and the fact that $X_1 = N_H - X_2$, $x_3 + x_4 + x_5 = 1$, and $x_6 + x_7 = 1$, system (3) reduces to one of the following equivalent systems (4) and (5). The first one using X_2, x_3, x_5, and x_7 is given by:

$$
\left\{
\begin{array}{lcl}
\dfrac{d\,X_2}{dt} & = & t_{15}\,(N_H - X_2)\,N_S\,x_5 \; - r_{12}\,X_2, \\
\dfrac{d\,x_3}{dt} & = & b_3 - (t_{32}\,X_2 + t_{37}\,N_M\,x_7 + b_3)\;x_3, \\
\dfrac{d\,x_5}{dt} & = & r_{54}\;(1 - x_3 - x_5) - b_3\,x_5, \\
\dfrac{d\,x_7}{dt} & = & t_{65}\,N_S\,x_5\;(1 - x_7) - b_6\,x_7.
\end{array}
\right.
\tag{4}
$$

The second one using X_2, x_4, x_5, and x_7 is given by:

$$
\left\{
\begin{array}{lcl}
\dfrac{d\,X_2}{dt} & = & t_{15}\,(N_H - X_2)\,N_S\,x_5 \; - r_{12}\,X_2, \\
\dfrac{d\,x_4}{dt} & = & (t_{32}\,X_2 + t_{37}\,N_M\,x_7)\;(1 - x_4 - x_5) - (b_3 + r_{54})\,x_4, \\
\dfrac{d\,x_5}{dt} & = & r_{54}\,x_4 - b_3\,x_5, \\
\dfrac{d\,x_7}{dt} & = & t_{65}\,N_S\,x_5\;(1 - x_7) - b_6\,x_7.
\end{array}
\right.
\tag{5}
$$

What is noticeable in these models is that the state of snails and mammalians are not available for the measurement in so far as the only available information at time t is the value of the infected human population. This means that it is possible to detect through clinical signs the number of infected people at a given time t. The value of the infected human population can be then seen as the measurable output of the reduced model. If it is possible to have the value of the state at a given time t_0 then it is possible to compute states for all $t \geq t_0$ by integrating the differential equation with the initial condition.
Unfortunately, it is not often possible to measure the whole state at a given time therefore it is impossible to integrate the differential equation for one does not know the initial condition.

What we can have is partial information of the state and that this partial information being itself given by $X_2(t)$ the so called output of the system. To do so we have to show the way to use this partial information combined with the model itself in order to have a dynamical estimation of the real unknown state variable. This estimation will then be produced by an auxiliary dynamical system called an observer which uses the information $X_2(t)$ provided by the system (4).
In addition there are numerous means to deal with the synthesis of nonlinear observers. The most general method to tackle it is to use a "high gain method observer" when the functions of the variables are perfectly known in the dynamical model. This means is much more general than "the output injection model" developed in [11], [12], [7], [8], which is applied to a very special class of systems only.
If it happens that some functions of the variables are partially known in the dynamical model but bounded with a priori known bounds, we can define a bounded error observer giving $\hat{X}(t)$ with $\left|\hat{X}(t) - X(t)\right|$ bounded by a "reasonable" positive real constant (depending on the uncertainty), "reasonable" meaning that this constant is small with respect to the measurement errors as developed in [6].
This paper shows out first a high gain observer for a reduced non linear model of schistosomiasis as proposed by Allen ([1]). This "high-gain observer" method, has been initiated in [3], [2], [4]. However, the convergence of this kind of observers is difficult to prove (because of the global Lipshitz condition). So, we propose a simpler observer whose convergence analysis is studied. This nonlinear observer is designed requires no Lipschitz extension of functions and no change of coordinates for the system contrary to the high gain observer.
In the second part, we will present an interval observer design to handle the already mentioned uncertainties of the model parameters. The methodology of interval observers has already been studied using a theoretical framework [17] and [15], and interval observers have been developed for particular models [5], [18], and have been validated experimentally [15]. In these works the authors address conditions for stability of the interval observer.
The construction of an observer requires some properties of observability and requires essentially the existence of globally defined and globally Lipschitzian change of coordinates.
The paper is organized as follows: Section 2 will points out a simple observer design, section 3 highlight the question of formulation we have been confronted with before talking about the non linear high gain design and fourth section will tackle the guaranteed interval observer, finally the sections 4 and 5 will respectively been constituted of the simulation exercises and the conclusion.

2 A first observer for a schistosomiasis model

In this section we consider system (5). It can be proved that the compact set $\mathcal{D} = [0, N_H] \times [0,1] \times [0,1] \times [0,1]$ is a positively invariant set under the flow of system (5).
The measurable output is $y(t) = X_2(t)$. We shall prove that a simple candidate observer for system (5) on the set $\mathcal{D}$ is given by:

$$\left\{\begin{array}{lll} \dfrac{d\hat{X}_2}{dt} & = & t_{15}\,(N_H - \hat{X}_2)\,N_S\,\hat{x}_5 \; - r_{12}\,\hat{X}_2 \; + L_1\,(y - \hat{X}_2) \\ \dfrac{d\,\hat{x}_4}{dt} & = & \left(t_{32}\,\hat{X}_2 + t_{37}\,N_M\,\hat{x}_7\right)\,(1 - \hat{x}_4 - \hat{x}_5) - (b_3 + r_{54})\,\hat{x}_4, \\ \dfrac{d\,\hat{x}_5}{dt} & = & r_{54}\,\hat{x}_4 - b_3\,\hat{x}_5, \\ \dfrac{d\,\hat{x}_7}{dt} & = & t_{65}\,N_S\,\hat{x}_5\,\,(1 - \hat{x}_7) - b_6\,\hat{x}_7. \end{array}\right. \tag{6}$$

It is remarkable that the set $\mathcal{D} = [0, N_H] \times [0,1] \times [0,1] \times [0,1]$ is a positively invariant compact set for system (6).
This observer is simply a copy of system (5) plus a corrective term given by $L_1\,(y - \hat{X}_2)$. The number L_1 is a constant positive real number that will be chosen in order to ensure the convergence of the estimation error.
We will denote $x(t) = (X_2(t), x_4(t), x_5(t), x_7(t))$ the state vector of system (5), and $\hat{x}(t) = \left(\hat{X}_2(t), \hat{x}_4(t), \hat{x}_5(t), \hat{x}_7(t)\right)$ the state vector of the candidate observer (6). The estimation error is $e(t) = (e_2(t), e_4(t), e_5(t), e_7(t)) = x(t) - \hat{x}(t)$.
We shall make the following assumptions on the model parameters:

Assumption 2.1. $\dfrac{N_h t_{32} + N_M t_{37}}{b_3 + r_{54}} < 1, \quad \dfrac{r_{54}}{b_3} \leq \dfrac{1}{2}, \quad \dfrac{N_S\,t_{65}}{b_6} \leq 1$

Proposition 2.1. *Under the assumption 2.1, the system governed by (6) is an exponential observer for the system (5) for L_1 satisfying*

$$L_1 \geq \max\left(\frac{N_h\,N_S\,t_{15}\,t_{32}}{b_3 + r_{54}} - r_{12}, 0\right).$$

i.e, there exists a positive real number λ such that for all initial conditions $(\hat{x}(0), x(0)) \in \mathcal{D} \times \mathcal{D}$, one has $|\hat{x}(t) - x(t)| \leq e^{-\lambda\,t}\,|\hat{x}(0) - x(0)|$.

Proof. The estimation error $e(t) = (e_2(t), e_4(t), e_5(t), e_7(t)) = x(t) - \hat{x}(t)$ obeys the following differential equation:

$$\dot{e} = A_d\,e + f(x) - f(\hat{x}) \tag{7}$$

where

$$A_d = \begin{pmatrix} -L_1 - r_{12} & 0 & 0 & 0 \\ 0 & -b_3 - r_{54} & 0 & 0 \\ 0 & 0 & -b_3 & 0 \\ 0 & 0 & 0 & -b_6 \end{pmatrix}, f(x) = \begin{pmatrix} t_{15}\,N_S\,x_5\,(N_h - X_2) \\ (t_{32}\,X_2 + t_{37}\,N_M\,x_7)\,(1 - x_4 - x_5) \\ r_{54}\,x_4 \\ t_{65}\,N_S\,x_5(1 - x_7) \end{pmatrix}.$$

Let $P = \begin{pmatrix} \dfrac{1}{2\,L_1+2r_{12}} & 0 & 0 & 0 \\ 0 & \dfrac{1}{2b_3+2r_{54}} & 0 & 0 \\ 0 & 0 & \dfrac{1}{2b_3} & 0 \\ 0 & 0 & 0 & \dfrac{1}{2b_6} \end{pmatrix}$ and consider the following candidate Lyapunov function for the error equation (7):

$$V(e) = e^T\,P\,e$$

We can write:

$$f(x) - f(\hat{x}) = \int_0^1 \frac{\partial f}{\partial x}(sx + (1-s)\hat{x})ds\,e = R(e,\hat{x})\,e$$

The explicit expression of the matrix $R(e,\hat{x})$ is given in appendix A.
Therefore $\dot{e} = (A_d + R(e,\hat{x}))\,e$ and then the derivative of $V(e)$ with respect to time along the solutions of the estimation error equation is

$$\dot{V}(e) = e^T\left(P\,A_d + A_d^T\,P + P\,R(e,\hat{x}) + R(e,\hat{x})^T\,P\right)e.$$

Simple computations give

$$\begin{aligned}\dot{V}(e) = \quad & -e_2^2 - e_4^2 - e_5^2 - e_7^2 \\ & -\frac{t_{15}(e_5+\hat{x}_5)N_S}{L_1+r_{12}}\,e_2^2 - \left(\frac{t_{32}(e_2+\hat{X}_2)}{b_3+r_{54}} + \frac{t_{37}(e_7+\hat{x}_7)N_M}{b_3+r_{54}}\right)e_4^2 - \frac{t_{65}(e_5+\hat{x}_5)N_S}{b_6}\,e_7^2 \\ & +\frac{t_{15}N_S\left(N_h-\hat{X}_2\right)}{L_1+r_{12}}\,e_2e_5 + \left(\frac{r_{54}}{b_3} - \frac{t_{32}(e_2+\hat{X}_2)+t_{37}(e_7+\hat{x}_7)N_M}{b_3+r_{54}}\right)e_4e_5 \\ & +\frac{t_{65}(1-\hat{x}_7)N_S}{b_6}\,e_5e_7 - \frac{(\hat{x}_4+\hat{x}_5-1)\,(e_2t_{32}+e_7t_{37}N_M)}{b_3+r_{54}}\,e_4\end{aligned}$$

$$\begin{aligned}\dot{V}(e) = \quad & -e_2^2\left(\frac{t_{15}x_5N_S}{L_1+r_{12}}+1\right) - e_4^2\left(\frac{t_{37}x_7N_M+t_{32}X_2}{b_3+r_{54}}+1\right) - e_5^2 - e_7^2\left(\frac{t_{65}x_5N_S}{b_6}+1\right) \\ & -\frac{t_{32}(\hat{x}_4+\hat{x}_5-1)}{b_3+r_{54}}\,e_2e_4 + \frac{t_{15}N_S\left(N_h-\hat{X}_2\right)}{L_1+r_{12}}\,e_2e_5 + \left(\frac{r_{54}}{b_3} - \frac{t_{37}x_7N_M+t_{32}X_2}{b_3+r_{54}}\right)e_4e_5 \\ & -\frac{t_{37}N_M(\hat{x}_4+\hat{x}_5-1)}{b_3+r_{54}}e_4e_7 + \frac{t_{65}(1-\hat{x}_7)N_S}{b_6}\,e_5e_7\end{aligned}$$

The expression of $\dot{V}$ can be written:

$$\begin{aligned}\dot{V}(e) = \quad & -(a_2+1)\,e_2^2 - (a_4+1)\,e_4^2 - e_5^2 - (a_7+1)\,e_7^2 \\ & -b_{24}\,e_2e_4 + b_{45}\,e_4e_5 - b_{47}\,e_4e_7 + b_{57}\,e_5e_7 + b_{25}\,e_2e_5,\end{aligned}$$

with: $a_2 = \dfrac{x_5N_st_{15}}{L_1+r_{12}}$; $\;a_4 = \dfrac{X_2t_{32}+x_7N_Mt_{37}}{b_3+r_{54}}$; $\quad a_7 = \dfrac{x_5\,N_st_{65}}{b_6}$;

$$b_{24} = \frac{(-1+\hat{x}_4+\hat{x}_5)\,t_{32}}{b_3+r_{54}};\;\; b_{45} = \frac{r_{54}}{b_3} - \frac{X_2 t_{32} + x_7 N_M t_{37}}{b_3+r_{54}};\;\; b_{47} = \frac{(-1+\hat{x}_4+\hat{x}_5) N_M\, t_{37}}{b_3+r_{54}};$$

$$b_{57} = \frac{(1-\hat{x}_7)\,N_s t_{65}}{b_7};\;\; b_{25} = \frac{\left(-\hat{X}_2+N_h\right) N_s t_{15}}{L_1+r_{12}}.$$

The derivative of $V(e)$ can be seen as a quadratic form in e_i. Applying the Gauss-Lagrange reduction to this quadratic form leads to:

$$-\dot{V}(e) = (1+a_2)\;(e_2+F_2(e_4,e_5))^2 + l_1\;(e_4+F_4(e_5,e_7))^2 + l_2\;(e_5+F_5(e_7))^2 + l_3\,e_7^2$$

where $l_1,\ l_2,\ l_3$ are functions of the model parameters, and the F_i are linear forms in their arguments. The exact expressions of the l_i and the F_i are given in Appendix A.
In Appendix A we show that if the parameters satisfy Assumption 2.1, then it is possible to choose a gain L_1 in such a way that all the l_i are positive. This proves that $\dot{V}$ is negative definite which ends the proof. □

2.1 A high gain observer for a schistosomiasis model

For a comparison purpose, we construct a high-gain observer for our system using the techniques developed in [3] and in [4]. The high-gain observer construction involves some complicated computations (see for instance [2], [3], [2],[9]). Since systems (4) and (5) are equivalent, we shall use system (4) which is more adapted to the high-gain observer construction.
Let us denote by $x(t) = (X_2(t), x_3(t), x_5(t), x_7(t))$ the state vector of system (4), f the vector field defining the dynamics of the system (4), and h the output function, that is $y(t) = h(x(t)) = X_2(t)$, and

$$f = \begin{pmatrix} t_{15}\,(N_H - X_2)\,N_S\,x_5 \ - r_{12}\,X_2 \\ -\,(t_{32}\,X_2 + t_{37}\,N_M\,x_7 + b_3)\ x_3 + b_3 \\ r_{54}\ (1 - x_3 - x_5) - b_3\,x_5 \\ t_{65}\,N_S\,x_5\ (1 - x_7) - b_6\,x_7 \end{pmatrix}.$$

To construct a high-gain observer for (4), one has to perform a change of coordinates in order to write the system in a simpler form. This usually done by using the output function together with its time derivative.
Let Φ be the function $\Phi : \mathring{\mathcal{D}} \to \mathbb{R}^4$ ($\mathring{\mathcal{D}}$ is the interior of $\mathcal{D}$) defined as follows:

$$\Phi(x) = \begin{pmatrix} h(x) \\ \\ L_f h(x) \\ \\ L_f^2 h(x) \\ \\ L_f^3 h(x) \end{pmatrix},$$

where L_f denotes the Lie derivative operator with respect to the vector field f. Thus

$$\Phi(x) = \begin{pmatrix} X_2 \\ \\ -r_{12}X_2 + N_S t_{15}\,(N_H - X_2)\,x_5 \\ \\ \begin{array}{c} N_S t_{15}\,(N_H - X_2)\,(r_{54}\,(1 - x_3 - x_5) - b_3 x_5) \\ +\,(-r_{12} - N_S t_{15} x_5)\,(-r_{12}X_2 + N_S t_{15}\,(N_H - X_2)\,x_5) \end{array} \\ \\ \begin{array}{c} (-r_{54} + r_{54}x_3 + b_3 x_5 + r_{54}x_5) \\ (N_S t_{15}\,(b_3\,(N_H - X_2) + N_H\,(r_{12} + r_{54} + 2N_S t_{15} x_5) - X_2\,(2r_{12} + r_{54} + 2N_S t_{15} x_5))) \\ -\,(r_{12}X_2 - N_H N_S t_{15} x_5 + N_S t_{15} X_2 x_5) \\ \left(r_{12}^2 + 2N_S r_{12} t_{15} x_5 + N_S t_{15}\,(r_{54}\,(-1 + x_3 + x_5) + x_5\,(b_3 + N_S t_{15} x_5))\right) \\ +N_S r_{54} t_{15}\,(N_H - X_2)\,(-b_3 + b_3 x_3 + t_{32} X_2 x_3 + N_M t_{37} x_3 x_7) \end{array} \end{pmatrix}.$$

The Jacobian of Φ can be written:

$$\frac{d\,\Phi}{d\,x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \\ -r_{12} - N_S\,t_{15}\,x_5 & 0 & N_S\,t_{15}\,(N_H - X_2) & 0 \\ \\ \alpha_0 & -N_S\,r_{54}\,t_{15}\,(N_H - X_2) & \alpha_1 & 0 \\ \\ \alpha_2 & \alpha_3 & \alpha_4 & N_M\,N_S\,r_{54}\,t_{15}\,t_{37}\,(N_H - X_2)\,x_3 \end{pmatrix},$$

where:

$$
\begin{aligned}
\alpha_0 &= (r_{12} + N_S\, t_{15}\, x_5)^{\,2} + N_S\, t_{15}\, (b_3 x_5 + r_{54}\, (-1 + x_3 + x_5)) \\
\alpha_1 &= -N_S\, t_{15}\, \left(b_3\, (N_H - X_2) + N_H\, (r_{12} + r_{54} + 2\, N_S\, t_{15}\, x_5) - X_2\, (2\, r_{12} + r_{54} + 2\, N_S t_{15}\, x_5)\right) \\
\alpha_2 &= N_S\, r_{54}\, t_{15}\, t_{32}\, (N_H - X_2)\, x_3 + N_S\, t_{15}\, (b_3 + 2\, r_{12} + r_{54} + 2\, N_S\, t_{15}\, x_5) + (-r_{12} - N_S t_{15} x_5) \\
&\quad (-b_3 x_5 - r_{54}\, (-1 + x_3 + x_5))\, \left((r_{12} + N_S\, t_{15} x_5)^{\,2} + N_S\, t_{15}\, (b_3 x_5 + r_{54}\, (-1 + x_3 + x_5))\right) \\
&\quad + N_S\, r_{54}\, t_{15}\, (b_3 - x_3\, (b_3 + t_{32}\, X_2 + N_M\, t_{37}\, x_7)) \\
\alpha_3 &= N_S\, r_{54}\, t_{15}\, (2\, b_3\, (N_H - X_2) + N_H\, (r_{12} + r_{54} + t_{32}\, X_2 + 3 N_S\, t_{15}\, x_5 + N_M\, t_{37}\, x_7)) \\
&\quad - N_S\, r_{54}\, t_{15}\, (X_2\, (3 r_{12} + r_{54} + t_{32} X_2 + 3\, N_S\, t_{15}\, x_5 + N_M\, t_{37}\, x_7)) \\
\alpha_4 &= b_3^2\, N_H\, N_S\, t_{15} + b_3\, N_H\, N_S\, r_{12}\, t_{15} + N_H\, N_S\, r_{12}^2\, t_{15} + 2\, b_3\, N_H N_S\, r_{54}\, t_{15} + N_H\, N_S\, r_{12}\, r_{54}\, t_{15} \\
&\quad + N_H\, N_S\, r_{54}^2\, t_{15} - 3\, N_H\, N_S^2\, r_{54}\, t_{15}^2 - b_3^2\, N_S\, t_{15}\, X_2 - 3\, b_3\, N_S\, r_{12}\, t_{15}\, X_2 - 3\, N_S\, r_{12}^2\, t_{15}\, X_2 \\
&\quad - 2\, b_3\, N_S\, r_{54}\, t_{15}\, X_2 - 3\, N_S\, r_{12}\, r_{54}\, t_{15}\, X_2 - N_S\, r_{54}^2\, t_{15}\, X_2 + 3\, N_S^2\, r_{54}\, t_{15}^2 + 3\, N_H\, N_S^2\, r_{54}\, t_{15}^2\, x_3 \\
&\quad - 3\, N_S^2\, r_{54}\, t_{15}^2\, X_2\, x_3 + 6\, b_3\, N_H\, N_S^2\, t_{15}^2\, x_5 + 4\, N_H\, N_S^2\, r_{12}\, t_{15}^2\, x_5 + 6\, N_H\, N_S^2\, r_{54}\, t_{15}^2\, x_5 \\
&\quad - 6\, b_3\, N_S^2\, t_{15}^2\, X_2\, x_5 - 6\, N_S^2\, r_{12}\, t_{15}^2\, X_2\, x_5 - 6\, N_S^2\, r_{54}\, t_{15}^2\, X_2\, x_5 + 3\, N_H\, N_S^3\, t_{15}^3\, {}_5^2 - 3\, N_S^3\, t_{15}^3\, X_2\, x_5^2.
\end{aligned}
$$

The determinant of $\dfrac{d\,\Phi}{d\,x}$ can be expressed by:

$$\Gamma(X_2, x_3, x_5) = N_M\, N_S^3\, r_{54}^2\, t_{15}^3\, t_{37}\, (N_H - X_2)^{\,3}\, x_3.$$

The Jacobian $\dfrac{d\,\Phi}{d\,x}$ is non nonsingular in the region $\mathring{\mathcal{D}}$ and moreover $\Phi(x)$ is one-to-one from $\mathring{\mathcal{D}}$ to $\Phi(\mathring{\mathcal{D}})$. So the map Φ is a diffeomorphism from $\mathring{\mathcal{D}}$ to $\Phi(\mathring{\mathcal{D}})$. This implies that the system (4) with the output $y(t) = X_2(t)$ is observable. In the news coordinates defined by $(z_1, z_2, z_3, z_4)^T = z = \Phi(x) = (h(x), L_f h(x), L_f^2 h(x), L_f^3 h(x))^T$, our system can be written in the canonical form as follows:

$$
\left\{
\begin{aligned}
\dot{z}(t) &= \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{A} z(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \Psi(z(t)) \end{pmatrix}, \\
y(t) &= z_1(t) = \underbrace{(1, 0, 0, 0)}_{C}\, z(t),
\end{aligned}
\right.
\tag{8}
$$

where: $\Psi(z) = L_f^4 h(\Phi^{-1}(z)) = L_f^4 h(x) = \psi(x)$.
The function ψ is smooth (it is a polynomial function of $x = (X_2, x_3, x_5, x_7)$) on the compact set $\mathcal{D}$. Hence, it is globally Lipschitz on $\mathcal{D}$. Therefore it can be extended by $\tilde{\psi}$, a Lipschitz function on $\mathbb{R}^4$ which satisfies $\tilde{\psi}(x) = \psi(x)$, for all $x \in \mathcal{D}$. In the same way we define $\tilde{\Psi}$ the Lipschitz prolongation of the function Ψ. So we have the following system (9) defined on the whole space

$\mathbb{R}^4$. The restriction of (9) to the domain $\mathcal{D}$ is the system (8):

$$\left\{\begin{array}{lcl} \dot{z} & = & A\,z + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(z) \end{pmatrix}, \\ y & = & C\,z. \end{array}\right. \tag{9}$$

According to [3], an exponential (high-gain) observer for system (9) is given by

$$\dot{\tilde{z}} = A\,\tilde{z} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(\tilde{z}) \end{pmatrix} + S^{-1}(\theta)\,C^T\,(y - C\,\tilde{z}), \tag{10}$$

where $S(\theta)$ is the solution of $0 = -\theta\,S(\theta) - A^T\,S(\theta) - S(\theta)\,A^T + C^T\,C$ and θ is large enough.

Here, $S(\theta) = \begin{pmatrix} \dfrac{1}{\theta} & -\dfrac{1}{\theta^2} & \dfrac{1}{\theta^3} & -\dfrac{1}{\theta^4} \\ -\dfrac{1}{\theta^2} & \dfrac{2}{\theta^3} & -\dfrac{3}{\theta^4} & \dfrac{4}{\theta^5} \\ \dfrac{1}{\theta^3} & -\dfrac{3}{\theta^4} & \dfrac{6}{\theta^5} & -\dfrac{10}{\theta^6} \\ -\dfrac{1}{\theta^4} & \dfrac{4}{\theta^5} & -\dfrac{10}{\theta^6} & \dfrac{20}{\theta^7} \end{pmatrix}$.

This observer is particularly simple since it is only a copy of system (9), together with a corrective term depending on θ. For the proof one can see [3].
An observer for the original system (4) can then be given by:

$$\left\{\begin{array}{l} \dot{\tilde{z}} = A\,\tilde{z} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tilde{\Psi}(\tilde{z}) \end{pmatrix} + S^{-1}(\theta)\,C^T\,(y - C\,\tilde{z}), \\ \hat{x}(t) = \Phi^{-1}(z(t)). \end{array}\right. \tag{11}$$

Or more simply a high-gain observer for the original system (4) can be given by:

$$\dot{\hat{x}} = \tilde{f}(\hat{x}) + \left[\frac{d\,\Phi}{d\,x}\right]^{-1}_{x=\hat{x}} \times S(\theta)^{-1}\,C^T(y - h(\hat{x})). \tag{12}$$

However, the set $\mathcal{D}$ which is positively invariant for system (4) is not necessary positively invariant for the observer (12), and $\Phi(\mathcal{D})$ is not positively invariant for the observer (10). Therefore the expressions $\left[\frac{d\,\Phi}{d\,x}\right]^{-1}_{x=\hat{x}}$ and $\Phi^{-1}(z(t))$ are not well defined in general.
If there exists $\tilde{\Phi}$ a prolongation of the diffeomorphism Φ to the whole space $\mathbb{R}^4$, that is $\tilde{\Phi}$ is a diffeomorphism from $\mathbb{R}^4$ to $\mathbb{R}^4$ whose restriction to $\mathring{\mathcal{D}}$ is Φ, then it will be sufficient to replace Φ by $\tilde{\Phi}$ in (11) and (12) and so all the expressions will be well defined. However, for our system such a prolongation does not exist since $\frac{d\,\Phi}{d\,x}$ is singular on the set $\{X_2 = N_M\} \cup \{x_3 = 0\}$. So instead

of working on $\mathcal{D}$, we have to consider first a set $\mathcal{D}_\epsilon \subset \mathcal{D}$ given by $\mathcal{D}_\epsilon = \mathcal{D} \cap \{X_2 < N_H - \epsilon, x_3 > \epsilon\}$. The positive number ϵ has to be chosen in such away that $\mathcal{D}_\epsilon$ is positively invariant for system (4). On $\{X_2 = N_H - \epsilon\}$, we have

$$\begin{aligned}\frac{d\,X_2}{dt} \leq \quad & t_{15}\,\epsilon\,(N_H - X_2)\,N_S\,x_5 \ - r_{12}\,(N_H - \epsilon\,) \\ & \leq \epsilon(t_{15}\,N_S + r_{12}) - r_{12}\,N_H.\end{aligned}$$

On $\{x_3 = \epsilon\}$, we have

$$\frac{d\,x_3}{dt} \geq \quad b_3 - (t_{32}\,N_H + t_{37}\,N_M + b_3)\ x_3.$$

So we take $\epsilon \leq \min\{\dfrac{r_{12}\,N_H}{t_{15}\,N_S + r_{12}}, \dfrac{b_3}{t_{32}\,N_H + t_{37}\,N_M + b_3}\}$.

Incidentally, even if in the domain $\mathcal{D}_\epsilon$ we can extend the diffeomorphism Φ it is difficult to find the explicit form. And then simulations will be done without extending the diffeomorphism Φ.

3 Design of the interval observer for a schistosomiasis model

The logic of interval observers is to generate estimated bounds that are caused by a lack of reliability in the models of measurements [17], [15]. Here, we intend to explore the possibility of designing interval observers in the case where transmission rate of the model (3) that are t_{15}, t_{32}, t_{37} and t_{65} remain partially known. Moreover, we seek to obtain an estimation even during the transients of the system that is to choose bounds that have been valid since the beginning. Given the uncertain bounds in the model, we are looking for dynamic ones to estimate the variables. This situation resembles many epidemiological models either by the lack of confidence in model parameters calibrated from experimental data, or because of models dynamical simplicity taking into account their complexity.
We will consider in this case that such parameters are bounded by a given positive number this means that that those measurements are no longer given by a single value but rather by an interval. The design is based on two points observers which help estimate in real time the lower and upper bound of the vectors state, given the following type of uncertain nonlinear systems.

$$(S):\begin{cases}\dot{x}(t) & = & Ax(t)+\psi(x,p),\\ y(t) & = & Cx(t),\\ x(t_0) & \in & [x_0]\wedge p\in[p],\end{cases}$$

where A is a matrix of dimension $n\times n$, $t\in[t_0,t_{n_T}]$, $[p]=[p^-,p^+]$ is a real interval vector of $\mathbb{R}^{n_p}$, $\psi\in\mathcal{C}^{k-1}(\mathcal{D}\times[p])$, $\mathcal{D}\times[p]\subseteq\mathbb{R}^{n+n_p}$ is an open set; n, m and n_p are the dimension of respectively the state vector x, the output vector y and the uncertain parameter vector p. The choice of the type of systems (S) is not restrictive since one can find reversible mappings for the state vector in order to transform a given system to a system of type (S), see [10]. We assume that measurements $y_m(t)$ are subject to an unknown but bounded, with known bound, additive error. Thus the feasible domain for measurements are given by the following boxes

$$\mathbb{Y}=[y_m(t)-b,y_m(t)+b],$$

where b is the vector of maximal measurement error.
We suppose that the input is uncertain with known bounds ψ^-, ψ^+ such that:

$$\psi^-\leq\psi\leq\psi^+,\forall t\in\mathbb{R}^+.$$

Remark: The inequalities applied to vectors must be considered term by term.
Under this assumption, we build two asymptotic observers.

Definition 3.1. *Let us consider the system (S). The pair of systems (S^-,S^+) with*

$$(S^-):\begin{cases}\dot{x}^-(t) & = & Ax^-(t)+B^-(\psi^-(t),\psi^+(t)),\\ x^-(t_0) & = & x_0^-,\end{cases}$$

$$(S^+):\begin{cases}\dot{x}^+(t) & = & Ax^+(t)+B^+(\psi^-(t),\psi^+(t)),\\ x^+(t_0) & = & x_0^+,\end{cases}$$

where $x_0^-\leq x_0\leq x_0^+$ is an interval estimator for the system (S) if for any compact set $\mathcal{D}_0\subset\mathcal{D}$, the coupled system (S,S^-,S^+) verifies for any initial conditions $x(t_0)\in\mathcal{D}_0$:

$$\forall t\geq t_0,\;\; x^-(t)\leq x(t)\leq x^+(t).$$

Function B^+ (respectively B^-) is such that:

$$B^-(\psi^-(t), \psi^+(t)) \leq B(\psi(t)) \leq B^+(\psi^-(t), \psi^+(t)).$$

We assume that the imprecisely known function ψ can be bounded by a lower and upper Lipschitz function. Moreover, there exists two known functions $\psi^-(\cdot)$ and $\psi^+(\cdot)$ built according to the bounds of $[p]$ and a known number $M < +\infty$ such that:

$$\left\{ \begin{array}{l} \forall p \in [p], \;\; \forall x \in \mathcal{D}, \\ \psi^-(x,p) \leq \psi(x,p) \leq \psi^+(x,p), \\ \|\psi^-(x,p) - \psi^+(x,p)\| \leq M. \end{array} \right. \tag{13}$$

Proposition 3.1. *([15])*
If there exists a gain K, a positive matrix, such as the non-diagonal elements of the matrix $(A - KC)$ are non-negative and Hurwitz with the condition (13) fulfilled, we can propose the interval observer for system (S) the same spirit as for the classical Luenberger approach, provided $x_0^- \leq x_0 \leq x_0^+$:

$$\left\{ \begin{array}{lcl} \dot{x}^+(t) & = & (A - KC)\, x^+(t) + \psi^+(x^-, x^+, p^-, p^+, u(t)) + K\, y_m^+(t), \\ \dot{x}^-(t) & = & (A - KC)\, x^-(t) + \psi^-(x^-, x^+, p^-, p^+, u(t)) + K\, y_m^-(t). \end{array} \right. \tag{14}$$

Remark: It is shown in [15] that the observation error remains positive and with $A - K\,C$ a Hurwitz matrix this error converges.
We consider our model whose dynamics are expressed in (4) and rewriting as follows for some convenience.

$$\left\{ \begin{array}{lcl} \dfrac{d\,X_2}{dt} & = & t_{15}\,(N_H - X_2)\, N_S\, x_5 \; - r_{12}\, X_2, \\ \dfrac{d\,x_4}{dt} & = & (t_{32}\, X_2 + t_{37}\, N_M\, x_7)\; (1 - x_4 - x_5) - (b_3 + r_{54})\, x_4, \\ \dfrac{d\,x_5}{dt} & = & r_{54}\, x_4 - b_3\, x_5, \\ \dfrac{d\,x_7}{dt} & = & t_{65}\, N_S\, x_5\; (1 - x_7) - b_6\, x_7. \end{array} \right. \tag{15}$$

We write system (15) in the typical form of (S) where

$$A = \begin{pmatrix} -r_{12} & 0 & 0 & 0 \\ 0 & -(b_3 + r_{54}) & 0 & 0 \\ 0 & r_{54} & -b_3 & 0 \\ 0 & 0 & 0 & -b_6 \end{pmatrix}$$

and

$$\psi = \begin{pmatrix} t_{15}\,(N_H - X_2)\, N_S\, x_5 \\ (t_{32} X_2 + t_{37} N_M x_7)\,(1 - x_4 - x_5) \\ 0 \\ t_{65} N_{\mathrm{S}} x_5\,(1 - x_7) \end{pmatrix}$$

and $C = (1, 0, 0, 0)$.

We assume that the transmission rates t_{15}, t_{32}, t_{37} are unknown but belong to the following intervals $[t_{15}^-, t_{15}^+]$, $[t_{32}^-, t_{32}^+]$, $[t_{38}^-, t_{38}^+]$. We denote $p = (t_{15}, t_{32}, t_{37})$. The initial state variables are also unknown but within the interval vector.
All the components $\psi(\cdot)$ of the vector function are Lipschitz on $\mathcal{D}$ with respect to the state vector for any $p \in [p]$. Then we can consider a Lipschitz extension of $\psi(\cdot)$ on $\mathbb{R}^4$ since $\mathcal{D}$ is positively invariant by the system (15), see for example [14], that we also denote by $\psi(\cdot)$. Moreover, there exists two known functions $\psi^-(\cdot)$ and $\psi^+(\cdot)$ built according to the bounds of $[p]$ such as

$$\psi^- = \begin{pmatrix} t_{15}^- \left(N_H - X_2\right) N_S\, x_5 \\ \left(t_{32}^- X_2 + t_{38}^- N_M x_7\right)\left(1 - x_4 - x_5\right) \\ 0 \\ t_{75}^- N_{\mathrm{Si}} x_5 \left(1 - x_7\right) \end{pmatrix}$$

and

$$\psi^+ = \begin{pmatrix} t_{15}^+ \left(N_H - X_2\right) N_S\, x_5 \\ \left(t_{32}^+ X_2 + t_{38}^+ N_M x_7\right)\left(1 - x_4 - x_5\right) \\ 0 \\ t_{75}^+ N_{\mathrm{Si}} x_5 \left(1 - x_7\right) \end{pmatrix}.$$

It is not hard to show that there exists a known number $M < +\infty$ such that the condition (13) fulfilled. For the observer gain $K = [l,\ 0,\ 0,\ 0]^T$, where l is a positive real number, the matrix

$$A - KC = \begin{pmatrix} -r_{12} - l & 0 & 0 & 0 \\ 0 & -b_3 - r_{54} & 0 & 0 \\ 0 & r_{54} & -b_3 & 0 \\ 0 & 0 & 0 & -b_6 \end{pmatrix}$$

is Hurwitz.
Therefore, given an interval estimate $([z_1^-, z_1^+], [z_2^-, z_2^+], [z_3^-, z_3^+], [z_4^-, z_4^+])^T$, the estimation in the basis (X_2, x_4, x_5, x_7) is given by

$$\left\{\begin{array}{lcl} \dot{z}_1^+ & = & -(r_{12} + l)\, z_1^+ + t_{15}^+ \left(N_H - z_1^+\right) z_3^+ N_{\mathrm{Si}} + l\, y_m^+(t), \\ \dot{z}_2^+ & = & -\left(b_3 + r_{54}\right) z_2^+ + \left(t_{32}^+ z_1^+ + t_{38}^+ N_M z_4^+\right)\left(1 - z_2^+ - z_3^+\right), \\ \dot{z}_3^+ & = & r_{54} z_2^+ - b_3 z_3^+ \\ \dot{z}_4^+ & = & -b_6 z_4^+ + t_{75}^+ N_{\mathrm{Si}}\, z_3^+ \left(1 - z_4^+\right), \\ \dot{z}_1^- & = & -(r_{12} + l)\, z_1^- + t_{15}^- \left(N_H - z_1^-\right) z_3^- N_{\mathrm{Si}} + l\, y_m^-(t), \\ \dot{z}_2^- & = & -\left(b_3 + r_{54}\right) z_2^- + \left(t_{32}^- z_1^- + t_{38}^- N_M z_4^-\right)\left(1 - z_2^- - z_3^-\right), \\ \dot{z}_3^- & = & r_{54} z_2^- - b_3 z_3^-, \\ \dot{z}_4^- & = & -b_6 z_4^- + t_{75}^- N_{\mathrm{Si}}\, z_3^- \left(1 - z_4^-\right). \end{array}\right. \tag{16}$$

4 Simulations

This part consists in showing out some simulation exercises to stress on the efficiency of the proposed observers of system (4) and system (5). The different snails and mammals populations are estimated via infected humans population measurements. In the first place the simpler observer design was implemented. Moreover, the reconstruction of the uncertain term is adequate and the population sizes estimations behave satisfactory along the simulations. For the simulation

of the high gain observer we extend the function f that defines the system (4) by continuity in order to make it globally Lipschitz on $\mathbb{R}^4$ in the following way: We denote $\tilde{f}$ the prolongation of f to $\mathbb{R}^4$ and the function π the projection on the domain $\mathcal{D}$ and we construct $\tilde{f} = f \circ \pi$. The extend function $\tilde{f}$ has the same Lipschitz coefficient as f. The projection π is defined as follows: for $x \in \mathbb{R}^4$, $\pi(x) = \overline{x}$ where $\overline{x} \in \mathcal{D}$ is such that $dist(x, \mathcal{D}) = \|x - \overline{x}\|$, i.e., $\overline{x}$ satisfies $\|x - \overline{x}\| = min_{u \in \mathcal{D}} \|u - x\|$. The initial values of state variables in all simulations concerning the high gain observer are $X_2(0) = 1600$, $x_3(0) = 0.4$, $x_5 = 0.3$, $x_7 = 0.5$, $\hat{u}_2(0) = 2000$, $\hat{x}_3(0) = 0.5$, $\hat{x}_5 = 0.2$, $\hat{x}_7 = 0.6$.
The parameters are: $t_{15} = 2, 23.10^{-2}$, $t_{37} = 2, 0.10^{-2}$, $t_{32} = 1, 05.10^{-2}$, $t_{65} = 1, 02.10^{-2}$, $r_{12} = 4, 47.10^{-1}$, $r_{54} = 2, 50.10^{-4}$, $b_3 = 11, 00.10^{-1}$, $d_6 = 1, 00.10^{-2}$, $d_3 = 8, 86.10^{-1}$, $d_6 = 1, 00.10^{-2}$, $c_{33} = 5, 11.10^{-5}$, $c_{36} = 5, 11.10^{-6}$, $c_{77} = 7, 00.10^{-6}$, $c_{66} = 1, 50.10^{-3}$, $c_{64} = 25, 11.10^{-4}$, $b_6 = 6, 60.10^{-2}$, $a_6 = b_6 - d_6$, $a_3 = b_3 - d_3$, $N_H = 5000$, $N_S = -\dfrac{(c_{66}\, a_3 - c_{36}\, a_6)}{(c_{33}\, c_{66} - c_{36}\, c_{64})}$, $N_M = \dfrac{b_7 - d_7}{c_{77}}$.
With these parameters, the gain chosen is $\theta = 2$ in all the simulations.
The simulation results are presented in figures 9 to 16 and illustrate the evolutions of the states variables and the estimated states with/without prolongation delivered by the high gain observer. Using the same parameters values, when we do not use the Lipschitz prolongation of the function f to the whole $\mathbb{R}^4$, the state estimation $\hat{x}$ computed by the observer tends to infinity in finite time. This actually happens in the beginning of the integration process. When the Lipschitz prolongation of the function f to the whole $\mathbb{R}^4$ is done, the convergence of the estimates delivered by the high gain observer is quite fast.

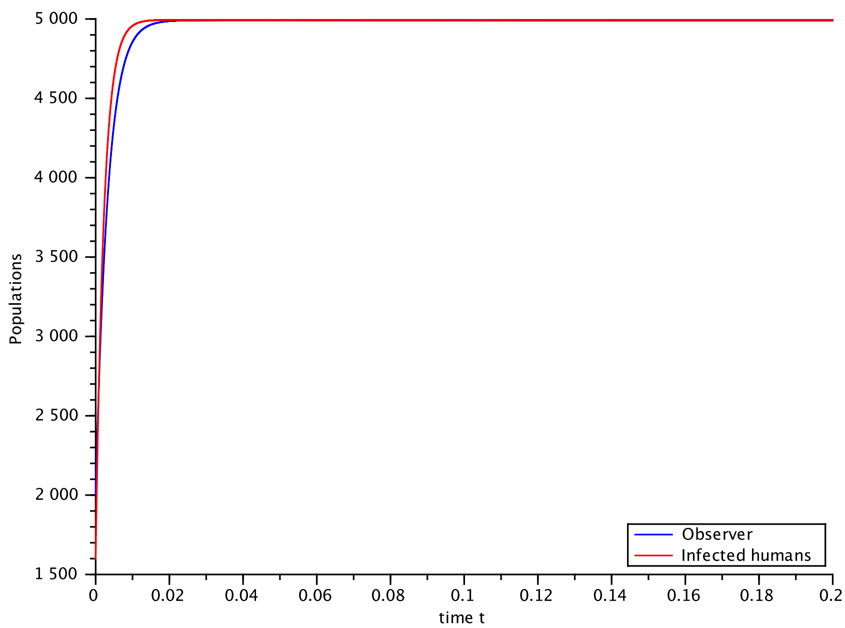

Figure 1: Time evolution of the number of infected humans $X_2(t)$ (red line) given by (5) and its estimate $\hat{X}_2(t)$ (blue line) delivered by the observer (6).

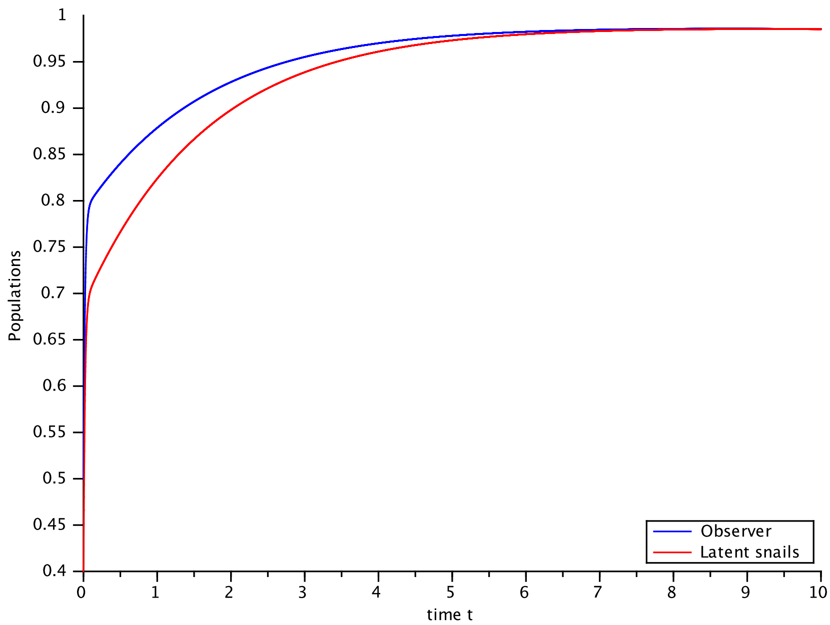

Figure 2: Time evolution of the number of latent snails $x_4(t)$ (red line) given by (5) and its estimate $\hat{x}_4(t)$ (blue line) delivered by the observer (6).

In order to test the robustness of the observer to noisy measurements, the measurements are vitiated by an additive Gaussian noise (with 30 as the signal-to-noise ratio per sample). In figures 13 to 16 the states variables and their estimates obtained using noisy data of X_2 are shown. However, such values are to be avoided since the estimators mate become noise sensitive. Simulations corresponding to free-noise and noisy data have been carried out with the same θ value. The chosen value is the one which provided the best compromise between fast convergence and well noise rejection. In the case of noisy measurements, the choice of relatively high values of θ are to be avoided since they amplify the noise and the obtained estimates may be unusable. The time histories of the estimates of the bounds on $X = (X_2, x_3, x_5, x_7)$, the states of the system (3), under the constraint that the transmission rates t_{32}, t_{37}, t_{65} are uncertain, are given in picture (Fig. 21 to 28). The partial unknown parameter vector is $p = [t_{15}, t_{32}, t_{37}, t_{65}]^T = [t_{15}^-, t_{15}^+] \times [t_{32}^-, t_{32}^+] \times [t_{38}^-, t_{38}^+] \times [t_{75}^-, t_{75}^+] = [1.23\,10^{-7}, 2.23\,10^{-7}] \times [0.05\,10^{-7}, 1.05\,10^{-7}] \times [1.00\,10^{-7}, 2.00\,10^{-7}] \times [0.02\,10^{-7}, 1.02\,10^{-7}]$. Model output is taken as $y(t) = X_2(t)$ and the maximal measurement error is $b = \mp 2\%\, y_m(\infty)$ where $y_m(\infty)$ is a nominal value. A gain vector is taken as $K^T = (3, 0, 0, 0)$. These last figures demonstrate that even with loose initial estimations on each bounds of unmeasured variables, we obtain estimates of the uncertainty intervals with a reasonable accuracy, the results keep on showing that the measurements are always inside the estimated bounds and the interval estimation converge to a box whose width depends itself on those measurements error bounds.

In the next simulations we consider a maximal measurement error $b = \mp 2\%\, y_m(\infty)$ with the same parameters of system as below:

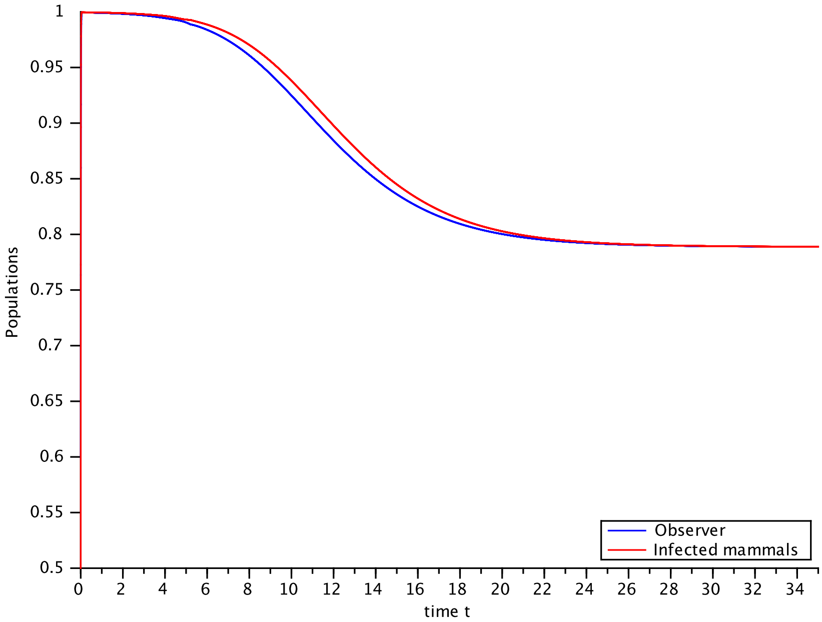

Figure 3: Time evolution of the number of infected mammals $x_7(t)$ (red line) given by (5) and its estimate $\hat{x}_7(t)$ (blue line) delivered by the observer (6).

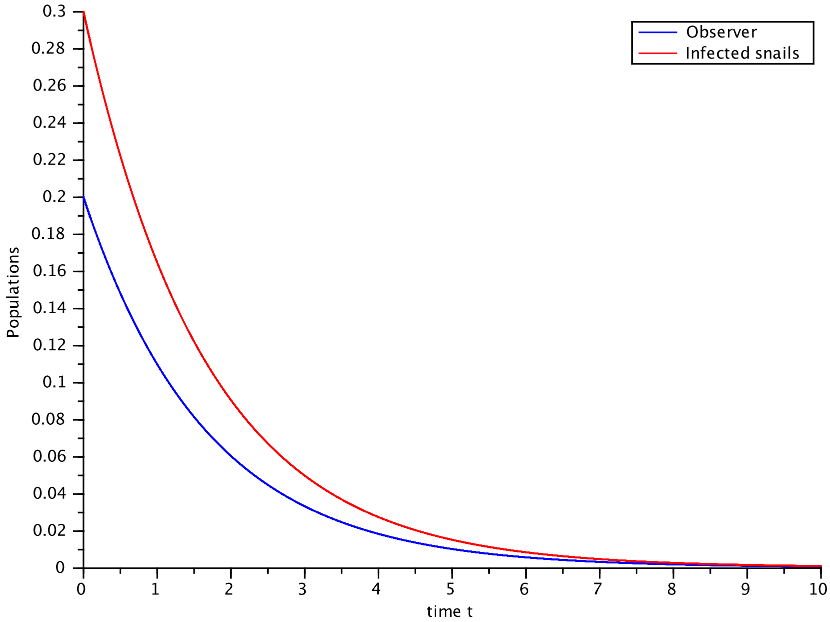

Figure 4: Time evolution of the number of infected snails $x_5(t)$ (red line) given by (5) and its estimate $\hat{x}_5(t)$ (blue line) delivered by the observer (6).

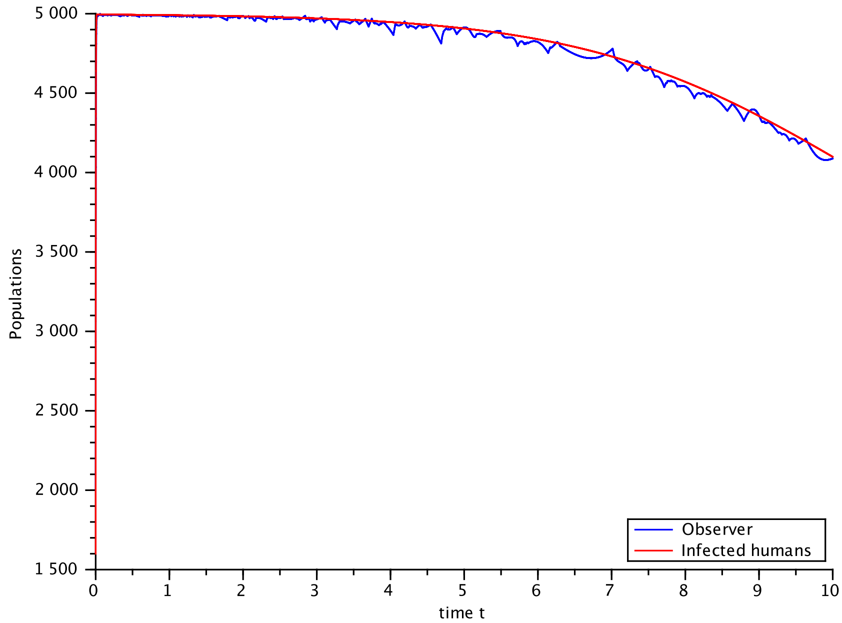

Figure 5: Infected humans population $X_2(t)$ (red line) and estimated state $\hat{X}_2(t)$ (blue line) versus time (t) with proposed observer (6), for system (5), with gaussian noise output X_2 measurements.

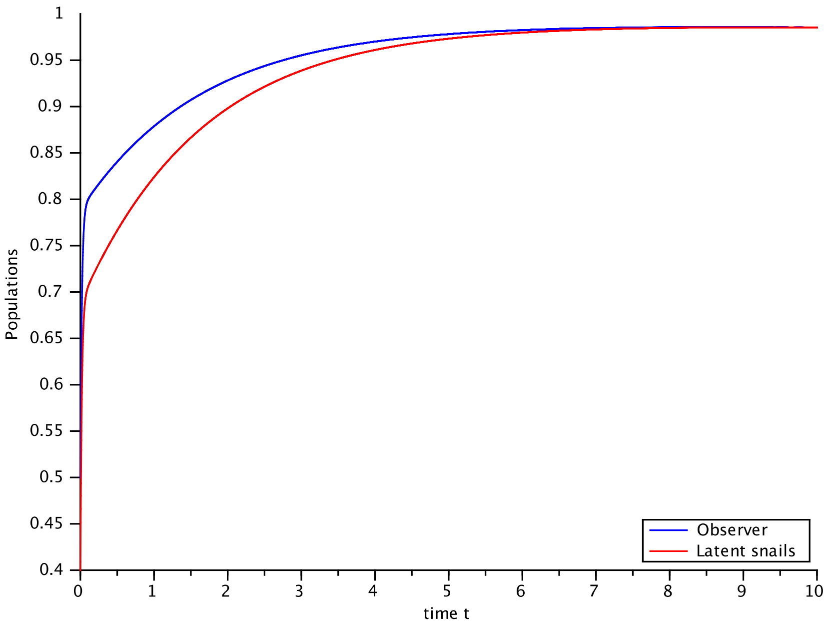

Figure 6: Latent snails population $x_4(t)$ (red line) and estimated state $\hat{x}_4(t)$ (blue line) versus time (t) with proposed observer (6), for system (5), with gaussian noise output X_2 measurements.

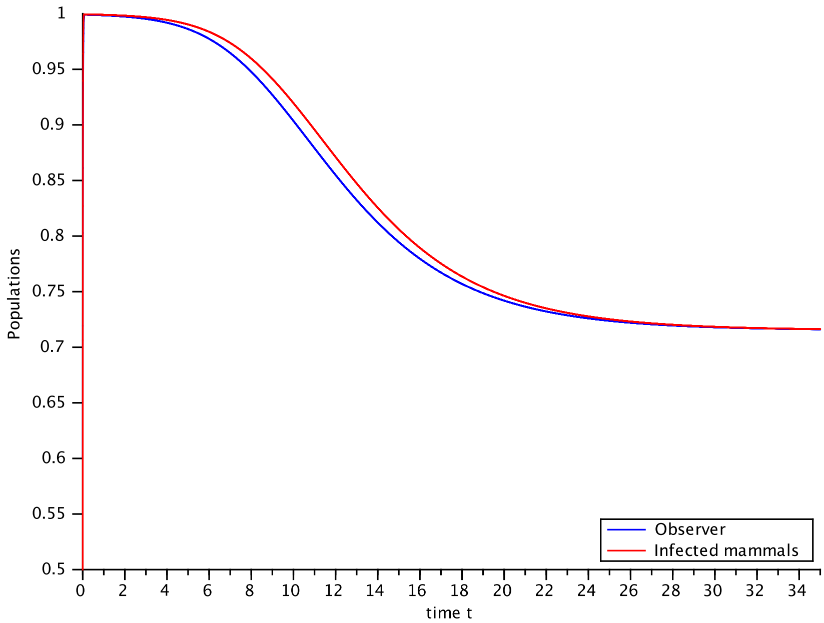

Figure 7: Infected mammals population $x_7(t)$ (red line) and estimated state $\hat{x}_7(t)$ (blue line) versus time (t) with proposed observer (6), for system (5), with gaussian noise output X_2 measurements.

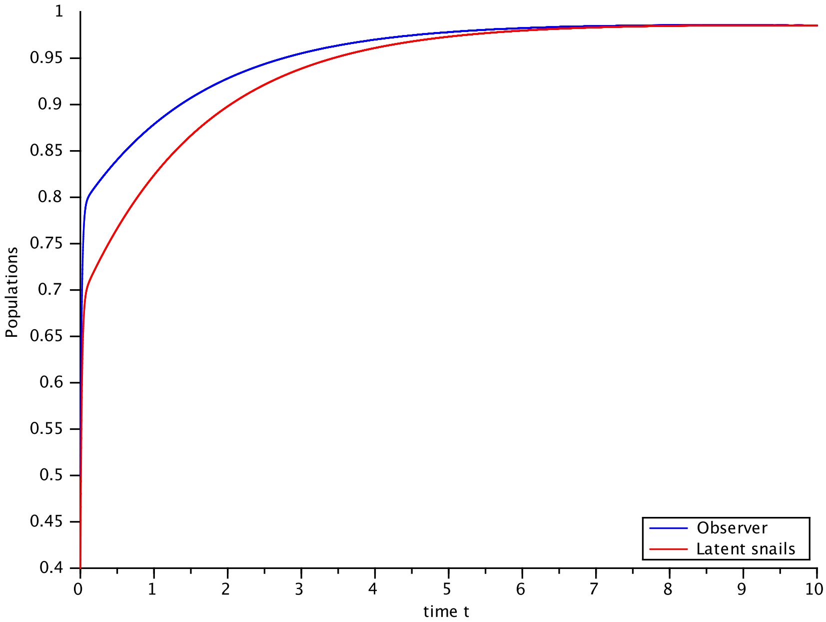

Figure 8: Infected snails population $x_5(t)$ (red line) and estimated state $\hat{x}_5(t)$ (blue line) versus time (t) with proposed observer (6), for system (5), with gaussian noise output X_2 measurements.

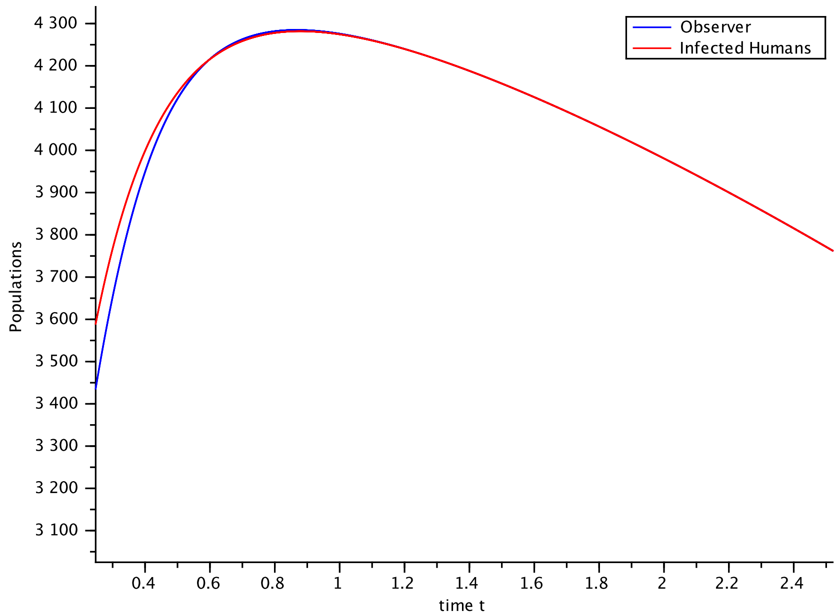

Figure 9: Simulation of system (4) with its observer:X_2 (red line) and its estimate $\hat{X}_2$(blue line) delivered by the high gain observer (12) when f is not extended

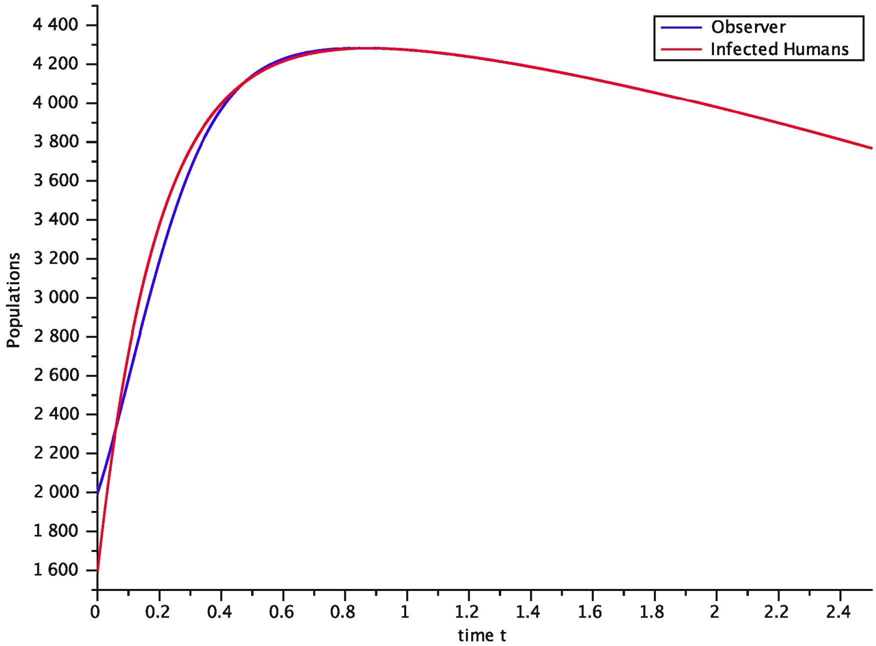

Figure 10: Simulation of system (4) with its observer: $X_2(t)$(red line) and its estimate $\hat{X}_2(t)$ (blue line) delivered by the high gain observer (12) when f is extended

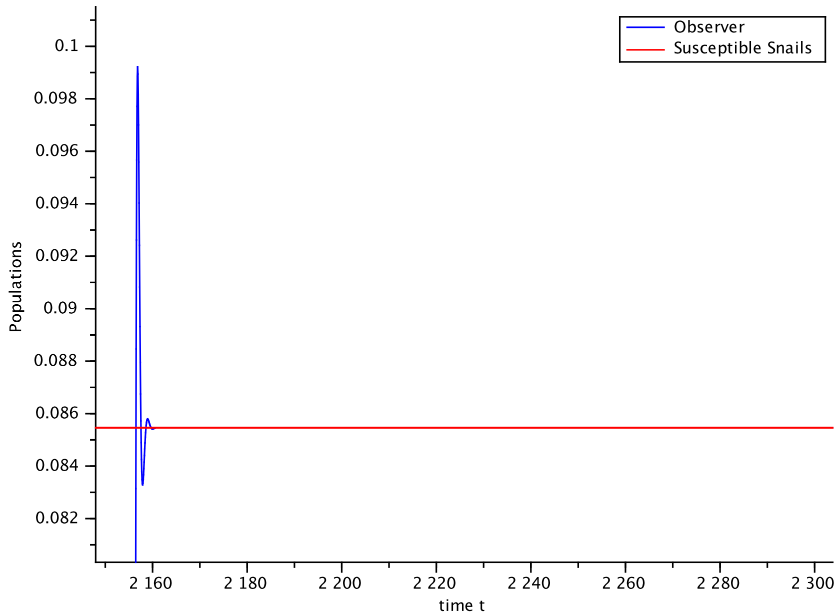

Figure 11: Simulation of system (4) with its observer: $x_3(t)$(red line) and its estimate $\hat{x}_3(t)$ (blue line) delivered by the high gain observer (12) when f is extended

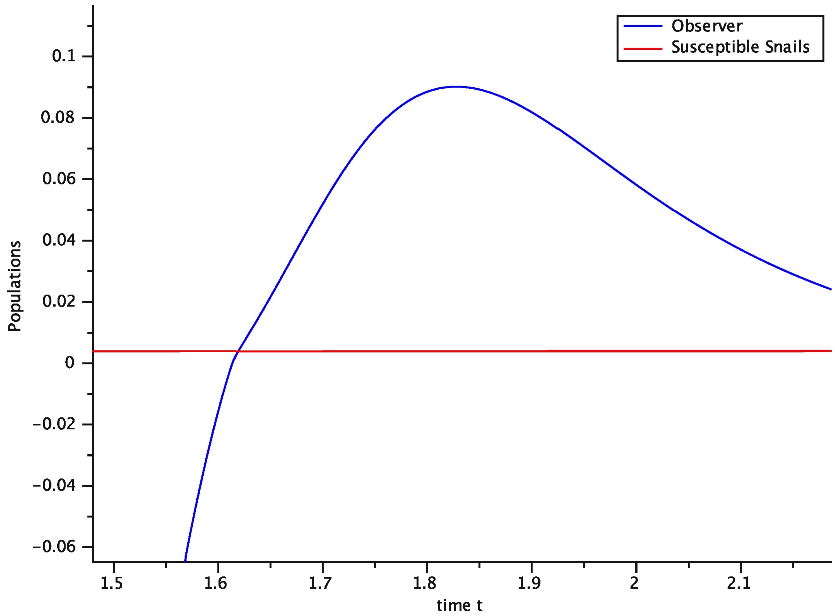

Figure 12: Simulation of system (4) with its observer: $x_3(t)$(red line) and its estimate $\hat{x}_3(t)$ (blue line) delivered by the high gain observer (12) when f is not extended

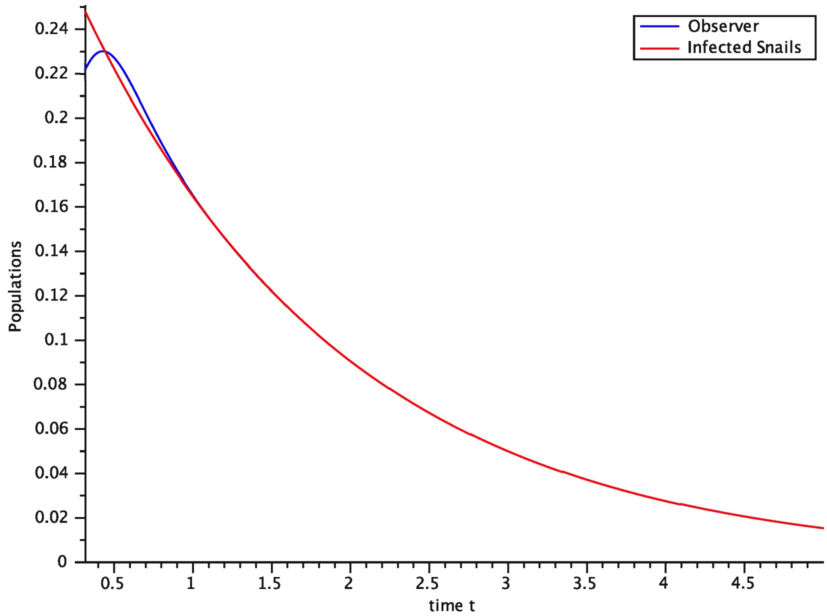

Figure 13: Simulation of system (4) with its observer: $x_5(t)$(red line) and its estimate $\hat{x}_5(t)$ (blue line) delivered by the high gain observer (12) when f is extended

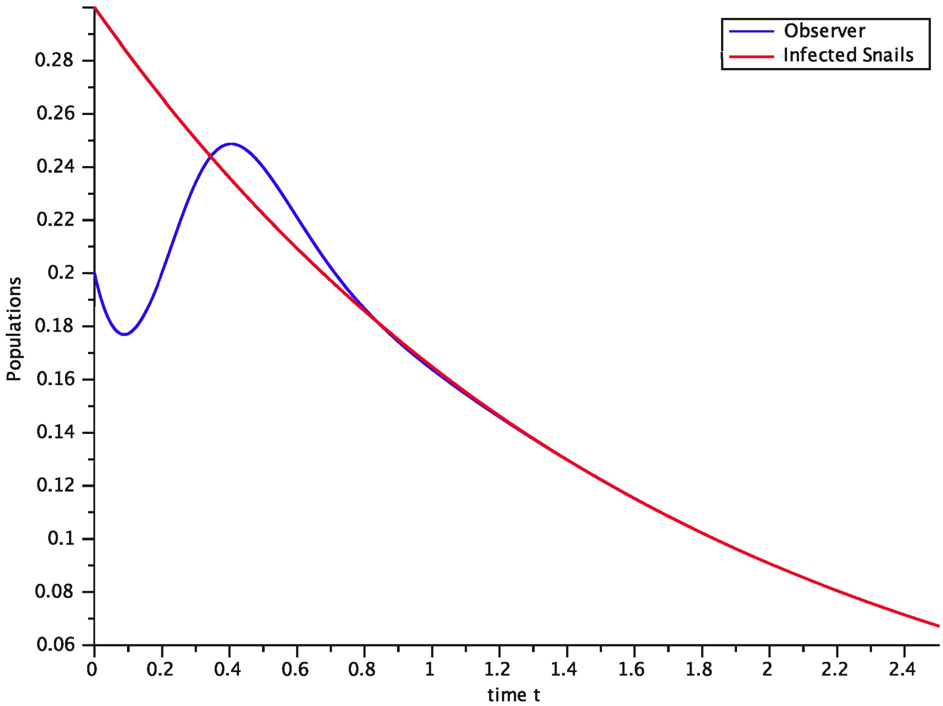

Figure 14: Simulation of system (4) with its observer: $x_5(t)$(red line) and its estimate $\hat{x}_5(t)$ (blue line) delivered by the high gain observer (12) when f is not extended

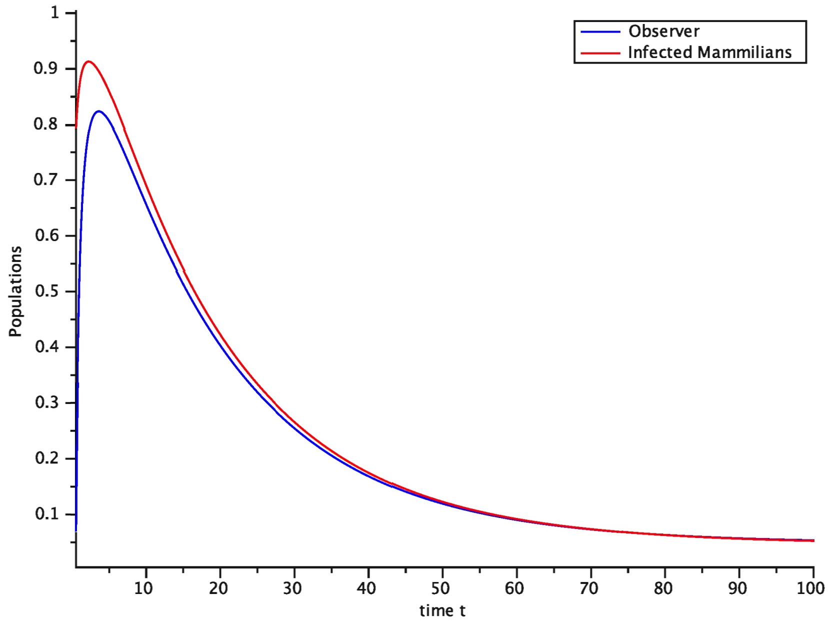

Figure 15: Simulation of system (4) with its observer: $x_7(t)$(red line) and its estimate $\hat{x}_7(t)$ (blue line) delivered by the high gain observer (12) when f is extended

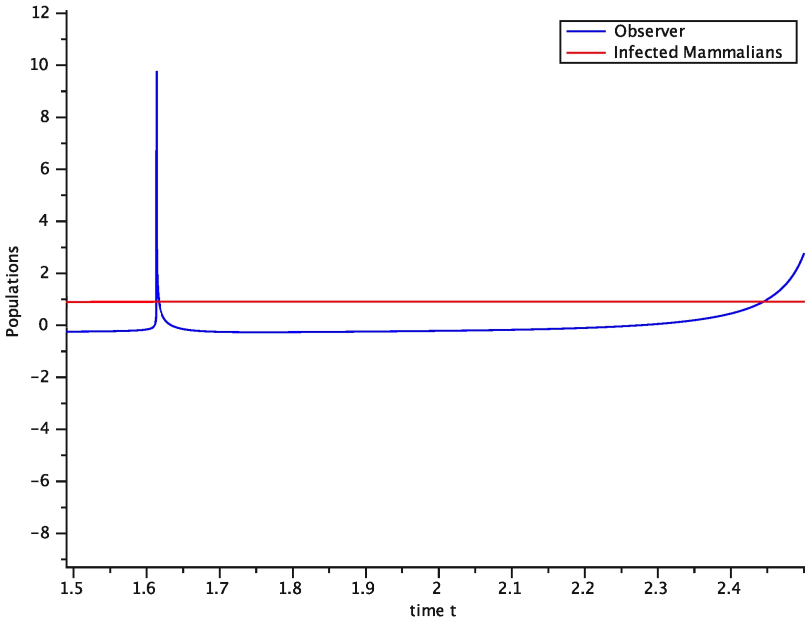

Figure 16: Simulation of system (4) with its observer: $x_7(t)$(red line) and its estimate $\hat{x}_7(t)$ (blue line) delivered by the high gain observer (12) when f is not extended

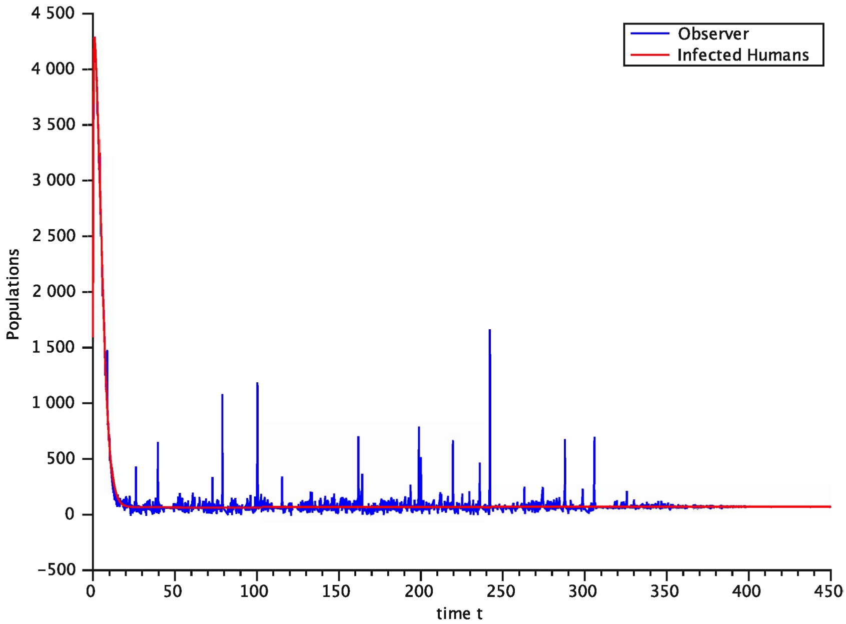

Figure 17: Simulation of system (4) and its observer (12) with gaussian noise output X_2 measurements when f is not extended: $X_2(t)$(red line) and its estimate $\hat{X}_2(t)$ (blue line) delivered by the high gain observer (12)

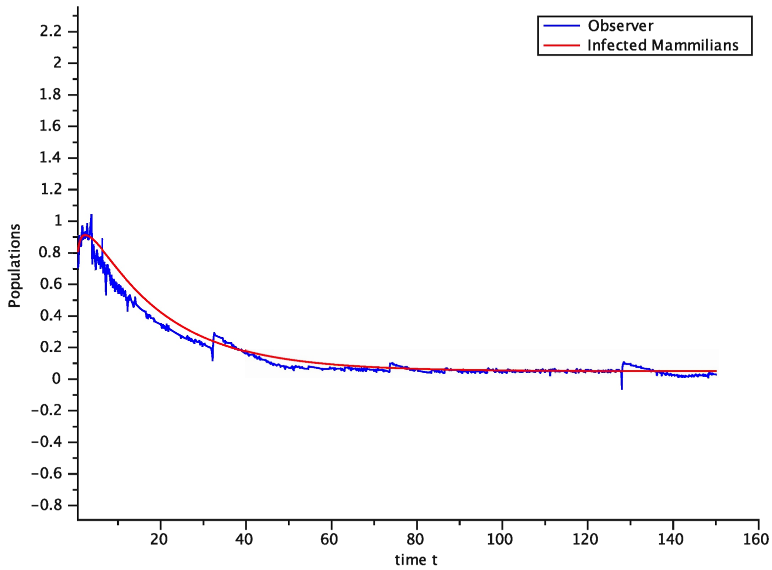

Figure 18: Simulation of system (4) and its observer with gaussian noise output X_2 measurements when f is not extended: $x_7(t)$(red line) and its estimate $\hat{x}_7(t)$ (blue line) delivered by the high gain observer (12)

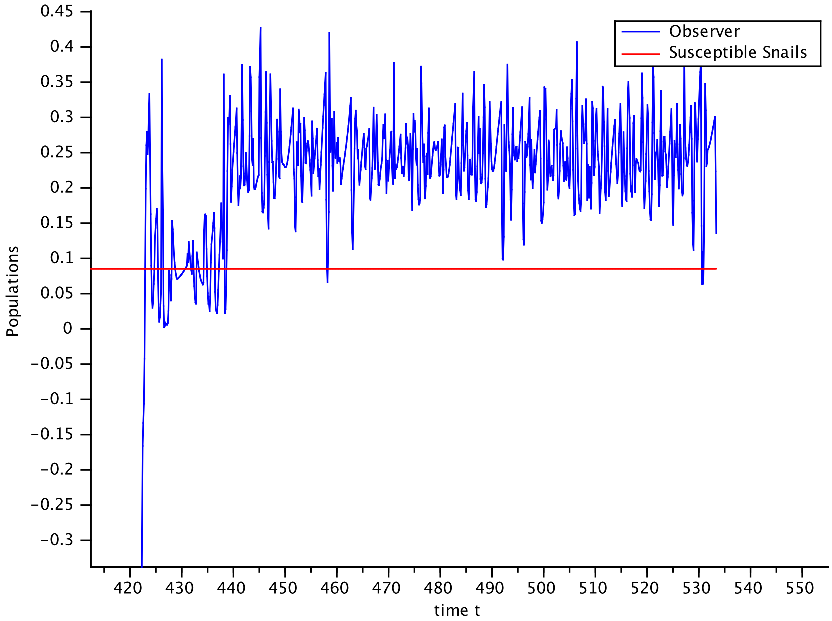

Figure 19: Simulation of system (4) and its observer (12) with gaussian noise output X_2 measurements when f is not extended: $x_3(t)$(red line) and its estimate $\hat{x}_3(t)$ (blue line) delivered by the high gain observer (12)

5 Concluding remarks

At the end of this work, let us recall that it dealt with non linear observers, designed and tested, using simulations to estimate the total population size of snails and mammalians; the study was besides covering noise free as well as noisy measurements. Then, to test the observers performance we had to use numerical simulations to demonstrate that the observer is robust when it is about modeling error and white Gaussian noise. Using the gain parameters θ adequate value, concerning the high gain observer, we finally obtained a satisfactory estimation of the real state. Actually, the high gain observer convergence is quite fast now and does no longer depend on the initial condition choice one can thus get a good estimation of the incommensurable real state some quickly.
Since the high gain observer convergence is difficult to prove and we have proposed a simple observer which convergence is obtained. We have also compared the two observers numerically. However the observers proposed before considering the model good enough, and the parameters values quite available, we choose to present another way of coping with uncertainties in the model, when estimating the variables though taking into account the outputs. Our basic idea was then simple: when given deterministic bounds on the uncertainties, we notice that one obtain deterministic guaranteed dynamic intervals containing the variables to estimate. The interval observer presented moreover very good convergence properties and by that was correctly predicting the dynamic bounds for the unmeasured variables.

Appendix A Proof Of Prop

Here we shall prove that the derivative of $V(e)$ is negative definite. Thanks to Gauss-Lagrange algorithm, $\dot{V}(e)$ can be written as follows:

$$\dot{V}(e) = -\Big((1+a_2)\ (e_2 + F_2(e_4,e_5))^2 + l_1\ (e_4 + F_4(e_5,e_7))^2 + l_2\ (e_5 + F_5(e_7))^2 + l_3\, e_7^2\Big).$$

where:

$$l_1 = 1 + a_4 - \frac{{b_{24}}^2}{4(1+\ a_2)};\ \ l_2 = 1 - \frac{{b_{25}}^2}{4(1+\ a_2)} - \frac{\left(\dfrac{b_{24}\,b_{25}}{4(1+\ a_2)} - \dfrac{b_{45}}{2}\right)^2}{1 + a_4 - \dfrac{b_{24}^2}{4(1+\ a_2)}}$$

$$l_3 = 1 + a_7 - \frac{b_{47}^2}{4\left(1+\ a_4 - \dfrac{{b_{24}}^2}{4(1+\ a_2)}\right)} - \frac{\left(\dfrac{\left(\dfrac{b_{24}b_{25}}{4+4\,a_2} - \dfrac{b_{45}}{2}\right)b_{47}}{2\left(1+a_4 - \dfrac{{b_{24}}^2}{4(1+a_2)}\right)} + \dfrac{b_{57}}{2}\right)^2}{1 - \dfrac{{b_{25}}^2}{4+4\,a_2} - \dfrac{\left(\dfrac{b_{24}b_{25}}{4+4\,a_2} - \dfrac{b_{45}}{2}\right)^2}{\left(1+a_4 - \dfrac{{b_{24}}^2}{4(1+\ a_2)}\right)}}$$

And:

$$F_2(e_4,e_5) = \frac{b_{24}}{2\,(a_2+1)}e_4 - \frac{b_{25}}{2\,(a_2+1)}\,e_5$$

$$\begin{aligned} F_4(e_5,e_7) &= \left(\frac{b_{24}\,b_{25}}{4(a_2+1)\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)} - \frac{b_{45}}{2\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)}\right)e_5 \\ &+ \frac{b_{47}}{2\left(-\dfrac{b_{24}^2}{4(a_2+1)} + a_4 + 1\right)}\,e_7 \\ &= \left(\frac{b_{24}\,b_{25}}{4\,(a_2+1)\,l_1} - \frac{b_{45}}{2\,l_1}\right)e_5 + \frac{b_{47}}{2\,l_1}\,e_7 \end{aligned}$$

$$F_5(e_7) = \frac{-\dfrac{b_{47}\left(\dfrac{b_{24}\,b_{25}}{4\,(a_2+1)}-\dfrac{b_{45}}{2}\right)}{2\left(-\dfrac{b_{24}^2}{4(a_2+1)}+a_4+1\right)}-\dfrac{b_{57}}{2}}{-\dfrac{\left(\dfrac{b_{24}b_{25}}{4(a_2+1)}-\dfrac{b_{45}}{2}\right)^2}{-\dfrac{b_{24}^2}{4(a_2+1)}+a_4+1}-\dfrac{b_{25}^2}{4(a_2+1)}+1}\,e_7$$

$$= \frac{1}{l_2}\left(-\frac{b_{47}\left(\dfrac{b_{24}\,b_{25}}{4\,(a_2+1)}-\dfrac{b_{45}}{2}\right)}{2\,l_1}-\frac{b_{57}}{2}\right)e_7$$

Let us prove that l_1, l_2 and l_3 are positives with assumption 2.1:

We recall the following quantities:

$$a_2 = \frac{x_5 N_s t_{15}}{L_1+r_{12}};\;\; a_4 = \frac{X_2 t_{32}+x_7 N_M t_{37}}{b_3+r_{54}};\;\; a_7 = \frac{x_5\,N_s t_{65}}{b_6};$$

$$b_{24} = \frac{(-1+\hat{x}_4+\hat{x}_5)\,t_{32}}{b_3+r_{54}};\;\; b_{45} = \frac{r_{54}}{b_3}-\frac{X_2 t_{32}+x_7 N_M t_{37}}{b_3+r_{54}};$$

$$b_{47} = \frac{(-1+\hat{x}_4+\hat{x}_5) N_M\, t_{37}}{b_3+r_{54}};\;\; b_{57} = \frac{(1-\hat{x}_7)\,N_s t_{65}}{b_7};\;\; b_{25} = \frac{\left(-\hat{X}_2+N_h\right)N_s t_{15}}{L_1+r_{12}}.$$

We have

$$a_7 \le \frac{N_s t_{65}}{b_7};\;\; -\frac{N_M t_{37}}{b_3+r_{54}} \le b_{47} \le \frac{N_M t_{37}}{b_3+r_{54}};\;\; -\frac{N_s t_{65}}{b_7} \le b_{57} \le \frac{N_s t_{65}}{b_7};$$

$$\frac{r_{54}}{2\,b_3}-\frac{1}{2}\frac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}} \le \frac{1}{2}\,b_{45} \le \frac{1}{2}\,\frac{r_{54}}{b_3};\;\; 0 \le \frac{{b_{25}}^2}{4(1+a_2)} \le \frac{1}{4}\left(\frac{N_h\,N_S\,t_{15}}{L_1+r_{12}}\right)^2$$

$$4 \le 4(1+\,a_2) \le 4(1+\frac{N_s t_{15}}{L_1+r_{12}}) \Rightarrow \frac{1}{4}\left(1+\frac{N_s t_{15}}{L_1+r_{12}}\right)^{-1} \le 4\,(1+\,a_2)^{-1} \le \frac{1}{4};$$

$$-\frac{1}{2}\frac{r_{54}}{b_3} \le -\frac{b_{45}}{2} \le -\frac{1}{2}\frac{r_{54}}{b_3}+\frac{1}{2}\,\frac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}$$

$$1 \le 1+a_4 \le 1+\frac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}$$

$$0 \leq \frac{{b_{24}}^2}{4(1+a_2)} \leq \frac{1}{4} \Rightarrow -\frac{1}{4} \leq -\frac{{b_{24}}^2}{4+4\,a_2} \leq 0$$

$$\frac{3}{4} \leq 1+a_4-\frac{{b_{24}}^2}{4(1+a_2)} \leq 1+\frac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}$$

$$-\frac{1}{4}\,\frac{N_h N_S t_{15} t_{32}}{(L_1+r_{12})\,(b_3+r_{54})} \leq \frac{b_{24}\,b_{25}}{4(1+a_2)} \leq \frac{1}{4}\,\frac{N_h N_S t_{15} t_{32}}{(L_1+r_{12})\,(b_3+r_{54})}$$

$$\frac{{b_{47}}^2}{4\left(1+a_4-\dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \leq \frac{1}{3}\left(\frac{N_M\,t_{37}}{b_3+r_{54}}\right)^2$$

$$-\frac{1}{2}\frac{r_{54}}{b_3} \leq \frac{b_{24}b_{25}}{4(1+a_2)}-\frac{1}{2}b_{45} \leq \frac{1}{4}\frac{N_h N_S t_{15} t_{32}}{L_1\,(b_3+r_{54})+r_{12}\,(b_3+r_{54})}-\frac{1}{2}\frac{r_{54}}{b_3}+\frac{1}{2}\frac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}$$

$$-\frac{\dfrac{1}{2}\dfrac{r_{54}}{b_3}}{\left(1+\dfrac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}\right)} \leq \frac{\dfrac{b_{24}b_{25}}{4(1+a_2)}-\dfrac{b_{45}}{2}}{\left(1+a_4-\dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \leq \frac{4}{3}\frac{N_h N_S t_{15} t_{32}}{4\,(b_3+r_{54})\,(L_1+r_{12})}-\frac{r_{54}}{2\,b_3}+\frac{N_h t_{32}+N_M t_{37}}{2(b_3+r_{54})}$$

$$\frac{\left(\dfrac{b_{24}b_{25}}{4(1+a_2)}-\dfrac{b_{45}}{2}\right)^2}{\left(1+a_4-\dfrac{{b_{24}}^2}{4(1+a_2)}\right)} \leq \frac{1}{3}\left(\frac{1}{2}\frac{N_h N_S t_{15}}{L_1+r_{12}}+\frac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}-\frac{r_{54}}{b_3}\right)^2$$

Since $\dfrac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}} < 1$, we have $\dfrac{t_{32}}{r_{54}+b_3} < 1$, we get

$$l_1 = 1+a_4-\frac{{b_{24}}^2}{4(1+a_2)} \geq 1-\frac{1}{4}\left(\frac{t_{32}}{b_3+r_{54}}\right)^2 > 0$$

$\star$ first case: Iff $b_{45} \geq 0$

We define
$X := \dfrac{N_h N_S t_{15} t_{32}}{(L_1+r_{12})\,(b_3+r_{54})}$ and $Y := \dfrac{r_{54}}{b_3}-\dfrac{N_h t_{32}+N_M t_{37}}{b_3+r_{54}}$

We choose $L_1 \geq 0$ so that $X \leq 1/2$.
we get

$$-\left(\frac{1}{2}\frac{r_{54}}{b_3}+\frac{X}{4}\right) \leq V_1 := \frac{b_{24}\,b_{25}}{4(1+a_2)}-\frac{1}{2}b_{45} \leq 0 \Rightarrow V_1^2 \leq \frac{1}{4}\left(\frac{r_{54}}{b_3}+\frac{X}{2}\right)^2$$

$$l_2 = \quad 1 - \frac{{b_{25}}^2}{4(1+a_2)} - \frac{\left(\frac{b_{24}b_{25}}{4(1+a_2)} - \frac{b_{45}}{2}\right)^2}{\left(1 + a_4 - \frac{{b_{24}}^2}{4(1+a_2)}\right)}$$

$$l_2 \geq \quad 1 - \frac{1}{4}\left(\frac{N_h\, N_S\, t_{15}}{L_1 + r_{12}}\right)^2 - \frac{1}{3}\left(\frac{r_{54}}{b_3} + \frac{X}{2}\right)^2$$

$$\geq \qquad \frac{3}{4} > 0$$

and since $b_{47} \leq 0$

$$V_2 := \frac{V_1\, b_{47}}{2\left(1 + a_4 - \frac{{b_{24}}^2}{4(1+a_2)}\right)} + \frac{b_{57}}{2} \leq \frac{V_1\, b_{47}}{2\,\frac{3}{4}} + \frac{b_{57}}{2} \leq \frac{2}{3}\left(\frac{X}{4} + \frac{r_{54}}{2\, b_3}\right)\frac{N_M t_{37}}{b_3 + r_{54}} + \frac{N_s t_{65}}{2\, b_7}$$

$$l_3 \geq 1 + a_7 - \frac{1}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)^2 - \frac{V_2^2}{l_2}.$$

Since $\dfrac{N_M\, t_{37}}{b_3 + r_{54}} \leq \dfrac{N_h t_{32} + N_M t_{37}}{b_3 + r_{54}} \leq \dfrac{r_{54}}{b_3}$

we get

$$l_3 \geq 1 + a_7 - \frac{1}{3}\left(\frac{r_{54}}{b_3}\right)^2 - \frac{V_2^2}{l_2}$$

$$l_3 \geq 1 + a_7 - \frac{1}{3}\left(\frac{r_{54}}{b_3}\right)^2 - 4\left(\frac{2}{3}\left(\frac{X}{4} + \frac{r_{54}}{2\, b_3}\right)\frac{r_{54}}{b_3} + \frac{N_s t_{65}}{2\, b_7}\right)^2$$

$$l_3 \geq a_7 + \frac{11}{12} - \frac{25}{48} > 0$$

$\star$ Second case: $\quad b_{45} \leq 0 \Rightarrow Y \leq b_{45} \leq 0$

We get

$$-\frac{1}{2}\frac{r_{54}}{b_3} \leq V_1 := \frac{b_{24}\, b_{25}}{4(1+a_2)} - \frac{1}{2}b_{45} \leq \frac{X}{4} - \frac{Y}{2}$$

Iff $|Y| \leq |\dfrac{r_{54}}{b_3}|$ then

$$|V_1| \leq |\frac{b_{24}\, b_{25}}{4(1+a_2)}| + |-\frac{1}{2}b_{45}| \leq \frac{X}{4} + \frac{r_{54}}{2\, b_3}$$

It follows that $0 \leq V_1^2 \leq \left(\dfrac{X}{4} + \dfrac{r_{54}}{2\, b_3}\right)^2$

So

$$l_2 \geq 1 - \frac{1}{4}\left(\frac{N_h\, N_S\, t_{15}}{L_1 + r_{12}}\right)^2 - \frac{1}{3}\left(\frac{X}{4} + \frac{r_{54}}{2\, b_3}\right)^2$$
$$l_2 \geq \frac{57}{64} > 0$$

$$|V_2| \leq |\frac{V_1\, b_{47}}{2\,\frac{3}{4}}| + |\frac{b_{57}}{2}| \leq \frac{2}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)\left(\frac{X}{4} + \frac{r_{54}}{2\, b_3}\right) + \frac{N_s t_{65}}{2\, b_7}$$
$$l_3 \geq 1 + a_7 - \frac{1}{3}\left(\frac{N_M t_{37}}{b_3 + r_{54}}\right)^2 - \frac{V_2^2}{l_2}.$$

Since $-\frac{r_{54}}{b_3} \leq Y \Rightarrow \frac{N_M t_{37}}{b_3 + r_{54}} \leq 2\frac{r_{54}}{b_3}$, we obtain $l_3 \geq 1 + a_7 - \frac{1}{3} - \frac{36}{57} > 0$.

Iff $|Y| \geq |\frac{r_{54}}{b_3}|$ then

$$|V_1| \leq |\frac{b_{24}\, b_{25}}{4(1 + a_2)}| + |-\frac{1}{2}b_{45}| \leq \frac{X}{4} + \frac{Y}{2}$$

It follows that $0 \leq V_1^2 \leq \left(\frac{X}{4} + \frac{Y}{2}\right)^2$

So

$$l_2 \geq 1 - \frac{1}{4}\left(\frac{N_h\, N_S\, t_{15}}{L_1 + r_{12}}\right)^2 - \frac{1}{3}\left(\frac{r_{54}}{b_3}\right)^2$$
$$l_2 \geq \frac{41}{48} > 0$$

$$|V_2| \leq |\frac{V_1\, b_{47}}{2\,\frac{3}{4}}| + |\frac{b_{57}}{2}| \leq \frac{2}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)\left(\frac{X}{4} + \frac{Y}{2}\right) + \frac{N_s t_{65}}{2\, b_7}$$
$$V_2^2 \leq \left(\frac{V_1\, b_{47}}{2\,\frac{3}{4}} + \frac{b_{57}}{2}\right)^2 \leq \left(\frac{2}{3}\left(\frac{N_M\, t_{37}}{b_3 + r_{54}}\right)\left(\frac{X}{4} + \frac{Y}{2}\right) + \frac{N_s t_{65}}{2\, b_7}\right)^2$$

So

$$l_3 \geq 1 + a_7 - \frac{1}{3}\left(\frac{N_M t_{37}}{b_3 + r_{54}}\right)^2 - \frac{V_2^2}{l_2}\; l_3 \geq 1 + a_7 - \frac{1}{12} - \frac{400}{1107} > 0.$$

And finally we give here the full expression of $R(e, \hat{x}) = \begin{pmatrix} R_{1,1} & R_{1,2} \\ R_{2,1} & R_{2,2} \end{pmatrix}$

where:

$$R_{1,1} = \begin{pmatrix} -\frac{1}{2}(e_5 + 2\hat{x}_5)N_S t_{15} & 0 \\ -\frac{1}{2}(e_4 + e_5 + 2(\hat{x}_4 + \hat{x}_5 - 1))t_{32} & \frac{1}{2}\left((-e_2 - 2\hat{X}_2)t_{32} - (e_7 + 2\hat{x}_7)N_M t_{37}\right) \end{pmatrix}$$

$$R_{1,2} = \begin{pmatrix} -\frac{1}{2}\left(e_2 + 2\hat{X}_2 - 2N_h\right)N_S t_{15} & 0 \\ \frac{1}{2}\left((-e_2 - 2\hat{X}_2)t_{32} - (e_7 + 2\hat{x}_7)N_M t_{37}\right) & -\frac{1}{2}(e_4 + e_5 + 2(\hat{x}_4 + \hat{x}_5 - 1))N_M t_{37} \end{pmatrix}$$

$$R_{2,1} = \begin{pmatrix} 0 & r_{54} \\ 0 & 0 \end{pmatrix}; \quad R_{2,2} = \begin{pmatrix} 0 & 0 \\ -\frac{1}{2}(e_7 + 2\hat{x}_7 - 2)N_S t_{65} & -\frac{1}{2}(e_5 + 2\hat{x}_5)N_S t_{65} \end{pmatrix}$$

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